II. *A Letter to* Dr. Mead, Coll. Med. Lond. & Soc. Reg. S. *concerning an Experiment, whereby it has been attempted to shew the falsity of the common Opinion, in relation to the force of Bodies in Motion.* By Henry Pemberton, M. D. R. S. S.

S I R,

PErusing the Learned *Polenus*'s Tract *de Castellis*, you were pleased to send me; I have found in it several curious Experiments, among which I reckon that of letting Globes of equal Magnitude, but of different weights, fall upon a yielding Substance, as Tallow, Wax, Clay or the like, from heights reciprocally proportional to the weights of the Globes. This Experiment engaged in particular my Attention, as it is brought with design to overturn one of the first Principles establish'd in Natural Philosophy. And the Knowledge I have of your great Esteem for that part of Science, emboldens me to trouble you with my Thoughts upon this Experiment; for I cannot by any means admit of the Deduction that is drawn from thence, that because the Globes make in this Experiment equal Impressions in the yielding Substance, therefore they strike upon it with equal force: whereby it is attempted to prove the Assertion of Mr. *Leibnitz*, that the force of the same Body in descending is proportional to the height from whence it falls; or, in all Motion, proportional to the Square of the Velocity, and not proportional to the Velocity it self, as is commonly thought. On the contrary, I think this very Experi-

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ment proves the great unreafonablenefs of Mr, *Leibnitz's* Notion.

I am furpriz'd, that fo careful a Writer as *Polenus* appears to be, from the accuracy wherewith he delivers his Experiments, fhould not rather fufpect his reafoning in an intricate Cafe, than thus contradict a Principle in Philofophy, that has been directly prov'd by a multitude of Experiments, in particular by thofe Sir *Ifaac Newton* recommends for that purpofe (*a*) ; and that is moreover abundantly eftablifh'd by its exact Agreement with all Obfervations ; as being the Principle upon which all appearances, hitherto obferved in the motion of Bodies, are accounted for by juft and undeniable Arguments; and we fhall find on Inquiry, that the prefent Cafe comes alfo under the fame Rule.

As the ufe of Experiments in Natural Philofophy is to difcover the Caufes of Things, by exhibiting more fimple Effects of thofe Caufes, than occur in the ordinary courfe of Nature; fo for this end it is neceffary, that our Argumentation upon Experiments be perfectly juft, otherwife they may lead us into Errors. The firft thing neceffary for making right Deductions from an Experiment, is to determine the proper ufe thereof; which I think in this before us is not rightly underftood. Certainly this Experiment of *Polenus* is much more fit to inform us of the Law, by which thefe yielding Subftances refift the motion of Bodies ftriking upon them, than to fhew the forces, with which Bodies ftrike ; for whatever thofe forces be, the Effects muft be very different, according to the Difference there may be in the Rule obferved by fuch refiftance.

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(*a*) Princip. Philof. Nat. *p.* 19.

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Now this Experiment fhews, that if two Globes in Motion bear againft equal Portions of the yielding Subftance, the Oppofition, that Subftance makes to the Motion of the Globes, will be the fame in both, however different the Velocities be, with which they move. This I fhall demonftrate as follows.

Let *A* and *B* be two Globes, equal in Magnitude, but of different Weights, which are equally immerfed into a yielding Subftance. Suppofe the Velocities, with which they move in their prefent Situation, to be reciprocally in the Subduplicate *ratio* of the Weights of the Globes ; that is, let the *ratio* of the Weight of the Globe *A* to the Weight of the Globe *B*, be Duplicate of the *ratio* of the Velocity of the Globe *B*, to the Velocity of the Globe *A*. Since therefore the *ratio* of the quantity of Motion in the Globe *A*, or of the force with which it moves, to the quantity of Motion in the Globe *B*, or to the force with which that Globe moves, is compounded of the *ratio* of the Weight of the Globe *A*, to the Weight of the Globe *B*, and of the *ratio* of the Velocity of the Globe *A*, to the Velocity of the other Globe *B*, the force, with which the Globe *A* moves, is to the force, with which the Globe *B* moves, as the Velocity of this Globe *B*, to the Velocity of the other Globe *A*. But if the fame Oppofition be made to the Motion of the Globes, when they bear upon equal Portions of the yielding Subftance, the Effect of that Oppofition, while the Globes enter farther into the Subftance by equal Spaces, will be proportional to the time, in which the Globes are moving thofe Spaces, or in which the Oppofition is made, if we confider thofe Spaces while nafcent or in their firft Origine; the Effect therefore of this Oppofition will be reciprocally proportional to the Velocity of each Globe : namely, the momentaneous lofs of force in the Globe

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*A* will be to the momentaneous loſs of force in the Globe *B*, as the Velocity of the Globe *B*, to the Velocity of the Globe *A* ; and the whole force of the Globe *A* has been found to bear the ſame *ratio* to the whole force of the Globe *B* ; conſequently theſe Globes, while they penetrate equal Spaces into the Subſtance, loſe parts of their force, which bear the ſame proportion to the whole : and therefore, if their Velocities be at any time reciprocally in the Subduplicate *ratio* of their Weights, ſo that the forces or degrees of Motion, with which they move, be reciprocally proportional to their Velocities, the forces, with which they preſs into the yielding Subſtance, at equal Indentures made in the Subſtance, will continue in the ſame Proportion ; and therefore upon the Theory of Reſiſtance here ſuppoſed, when the whole Force and Motion of both theſe Globes is entirely loſt, they will be plunged into the Subſtance at equal Depths.

Now whereas in the Experiment of *Polenus*, the Globes, falling from Heights reciprocally proportional to their Weights, ſtrike upon the yielding Subſtance with Velocities reciprocally in the Subduplicate proportion of their Weights, and the Effect is in all Caſes found to be, what is here deduced from the Theory of Reſiſtance, I have propoſed ; it is a ſufficient Confirmation of the Truth of this Theory.

Only here, Sir, I ought to obſerve to you, That I have ſuppoſed the Globes to be ſtopt by the whole Reſiſtance of the Subſtance, they move againſt ; although in ſtrictneſs they are ſtopt only by the exceſs of that Reſiſtance above the Action of Gravity upon them. But I have neglected the Conſideration of the Action of Gravity, that being but ſmall in Proportion to the Reſiſtance, as will appear from the Globes being much more ſpeedily ſtopt by this Reſiſtance, than they

they would be by the Action of Gravity, if its force were applied upwards ; for by that force alone, the Globes would not be ftopt, till they had meafured Spaces equal to the heights above the refifting Subftance, from whence they fell ; which heights bear a great Proportion to the depths, the Globes in this Experiment are immerfed into the yielding Subftance, as I have found upon trial.

Thus, if I miftake not, may be removed the Difficulty attending this Experiment. But as Mr. *Leibnitz's* Opinion is deduced from it by Means of this Axiom, that Effects are proportional to their Caufes ; fo that here the Effects being thought the fame, the Caufes are concluded to be fo likewife ; it will not be a-mifs to mention in this place an Experiment, where this Axiom may be more jnftly applied, than it can be in our prefent cafe, from which Experiment the received Opinion may be proved. This Experiment is mentioned by *Pol nus* from *Merfennus*, though tried by him fomewhat rudely ; but has been often fince made in the following manner. To one end of the Beam of a Ballance is hung a Weight, and from a proper height is let fall upon the other extremity of the Beam another Weight, which by ftriking thereon fhall raife the end, to which the Weight is fufpended, to fuch a height, as is juft fufficient to fet free a certain Spring. If then a different Weight be hung in the room of the former; the height, from whence the fallingWeight muft defcend,in order to raife the Extremity of the Ballance, to which this otherWeight is fufpended, to the fame height as before, that is, high enough to fet the forementioned Spring at liberty, is found to be fuch, that the Velocity, with which the falling Weight ftrikes upon the Ballance, in this latter Cafe, will be to its former Velocity, as the latter Weight

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to the former, *(a)* except only that, agreeably to what *Merſennus* himſelf remarked, when the Weight deſcends from great heights, an Altitude ſomewhat greater than this Rule implies is required to raiſe the other Weight as much as is deſired. But whether the bending of the Arm of the Ballance, when acted upon with a great force, or whether any increaſe of Friction in this Caſe, occaſion the Irregularity here mentioned, we need not ſtrictly enquire; for this Irregularity is yet leſs reconcileable with the new Opinion, than the regular Effects of the Experiment. Hence therefore we may ſee, that the very Method of reaſoning, which being applied erroneouſly, is ſuppoſed to prove Mr. *Leibnitz*'s Sentiment concerning the force of Bodies in Motion, will, when juſtly uſed, confirm the other Opinion in relation to that Matter.

But as I have aſſerted in the beginning of this Letter, that the very Experiment of *Polenus* is not only reconcileable to the common Doctrine of Motion, as I have now demonſtrated; but even that it does it ſelf make manifeſt the great unreaſonableneſs, if not the abſolute Abſurdity, of Mr. *Leibnitz*'s Opinion; it remains that I briefly make proof of this.

If two Globes *A* and *B*, of equal Magnitude but of different Weights, ſtriking on a yielding Subſtance with equal force, in every Caſe loſe all their Motion at equal Depths, it is neceſſary that at all times, during their Motion, they loſe equal Degrees of force, when they bear upon equal Portions of the Subſtance, in entring equal Spaces into the Subſtance. This will be eaſily ſeen from what has before been ſaid. Now whereas Mr. *Leibnitz* ſuppoſes the power of Gravity to

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(*a*) *Vid.* 's Graveſande Phyſic. Elem. Tom. I. p. 39.

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give to the fame falling Body Degrees of force proportional to the height from whence it falls; according to his Opinion, by the power of Gravity, equal Degrees of force are added in the descent of the fame Body through equal Spaces ; and in different Bodies descending through equal Spaces, the Degrees of force added will be as the quantity of Matter, or as the weight of each Body. Therefore while the Globes *A* and *B* penetrate equal nascent Spaces into the yielding Substance, by the Action of Gravity, were not that Action overcome by the Resistance of that Substance, additional Degrees of force would be communicated in such proportion, that the force added to the Globe *A*, would be to the force added to the Globe *B*, as the weight of the Globe *A*, to the weight of the Globe *B*, or in the Duplicate *ratio* of the Velocity of the Globe *B*, to the Velocity of the Globe *A*. But since the Globes lose the fame Degrees of force in entring equal Nascent Spaces into the yielding Substance, the Effect of the Opposition made by this Substance to the Motion of the Globes, during the time of their passing through such Nascent Spaces, will be both the taking from them that fame Degree of force, and moreover the additional force, which would otherwise have been given them by their own Gravity. But farther, the Opposition made to the motion of the Globe *A*, to the Opposition made to the motion of the Globe *B*, will be in the *ratio* compounded of the *ratio* of the Effect of the Opposition, the Substance makes to the motion of the Globe *A*, to the Effect of the Opposition, the Substance makes to the motion of the Globe *B*, and of the *ratio* of the Time, in which the Opposition is made against the latter Globe, to the Time in which it is made against the former ; which latter *ratio* is the
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fame with the *ratio* of the Velocity of the Globe *A*, to
the Velocity of the Globe *B*. But fince it is fhewn, that
the Effect of the Oppofition made by the yielding Sub-
ftance to thefe Globes is two-fold, and that one part
of the Effect of the Oppofition made to the motion of
the Globe *A*, is equal to one part of the Effect of the
Oppofition made to the motion of the Globe *B*; and
that another part of the Effect of the Oppofition
made to the motion of the Globe *A*, to another part
of the Effect of the Oppofition made to the motion
of the Globe *B*, is in the Duplicate *ratio* of the Velo-
city of the Globe *B*, to the Velocity of the Globe
*A*. One part of the Oppofition it felf made to the
motion of the Globe *A*, will be to one part of the
Oppofition againft the motion of the Globe *B*, as the
Velocity of the Globe *A*, to the Velocity of the Globe
*B*, and another part of the Oppofition to the motion of
the Globe *A*, to another part of the Oppofition to the
motion of the Globe *B*, will be as the Velocity of the
Globe *B*, to the Velocity of the Globe *A*. So that
when the Globes bear upon equal Portions of the yield-
ing Subftance, the Oppofition to their Motion will be
in part as the Velocity of the Globes, and in part
reciprocally as their Velocity. Hence, becaufe the re-
fifting Subftance is of an uniform Texture, the Oppo-
fition to the Motion of either of the Globes in its pre-
fent Situation, and when moving with its prefent Velo-
city, will be to the Oppofition it would meet with in
the fame Situation, if it moved with any other Veloci-
ty, partly as the prefent Velocity to that other Velo-
city, and partly as that other Velocity to the prefent.
But by that part of the Oppofition made againft the
motion of the Globe, which is directly as the Veloci-
ty, the Globe can never be wholly ftept; for upon
the ftopping of the Globe, that part of the Oppofi-
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tion to its Motion will likewife totally ceafe, and con-
fequently the Globe's Weight will carry it further
down, unlefs the other part of the Oppofition againſt
its Motion prevent it. But I fay again, neither can
this latter part of the Oppofition made to its Motion
be ever great enough to ftop the Globe; for the
Degree of this Oppofition being reciprocally as the
Velocity of the Globe, when the motion of the
Globe is wholly taken away, it will become infinite-
ly greater, than at any time, while the Globe is in
Motion; fo that when the Globe fhould be ftopt by
this part of the Oppofition made to its Motion, the
Oppofition to the Globe's Motion will become infinite-
ly great; infomuch, that no Degree of force whatever
could be able to impel the Globe further into the Sub-
ftance, but this can never come to pafs. Befides, it
is not neceffary to apply any fuch refined Argument
againſt this part of the Refiftance; it would be alone
fufficient to confider, how unreafonable a Suppofition
it is, that a Refiftance fhould increafe, when the Ve-
locity of the refifted Body decreafes.

Thus may this Experiment be made ufe of to inva-
lidate that very Opinion it is brought to fupport.
But another ufe may likewife be made thereof: For it
will ferve to illuftrate what the great Sir *Ifaac Newton*
has more than once hinted, that the Refiftance of
Fluids, which arifes from the Tenacity of their Parts,
decreafes in a lefs Proportion than the Velocity of the
refifted Body decreafes (*a*); for as this Refiftance bears
a great Analogy to the Refiftance of the yielding Sub-
ftances we have been here treating of, fo we have
found, that the Refiftance of thefe Subftances does

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(*a*) *Vid.* Philof. Nat. Princip. Math. Prop. 52. lib. 2. in Schol.
Opticks. Qu. 28. p. 339, 340.

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not much depend upon the Velocity of the Body, againſt which the Reſiſtance is applied.

And thus, Sir, we may ſee how all Experience con-ſpires in confirming and ſetting forth that ſtupendi-ous force of reaſoning, which has enabled our great Philoſopher moſt ſurprizingly to ſearch out, and di-ſtinguiſh the Springs of Natural Operations; a Work infinitely more difficult to accompliſh than even the great Improvements he has made in pure Mathematicks, which were previouſly neceſſary in order to his ſucceeding in his Searches after the Knowledge of Nature ; for in this laſt Purſuit he has given Proof, not only of a more unbounded Invention, than is required in the ſubtileſt Geometrical Speculations; but has alſo there diſcovered the greateſt Diſcernment, and moſt conſummate Judg-ment ; ſince in his Philoſophical Writings, he has never been once impoſed on by an Hypotheſis, nor by any other of the various Fallacies, which my Lord *Bacon* in his *Novum Organon*, has reckon'd up as the Cauſes, that had hindred the Improvement of the true Philoſophy.

But here I ſhall put an end to this long Letter ; for the Freedom of which, I know, I need make no Apology to you, Sir, of whoſe great Candour I have for ſome Years paſt been a conſtant Witneſs ; and as I have frequently admired, how amidſt the vaſt Em-ployments of your Profeſſion, you ſhould find time to purſue with ſo great Succeſs ſuch various ſorts of Learning ; ſo I have as often been delighted to ob-ſerve with what Benignity you receive thoſe, who have made any the leaſt part of uſeful Knowledge their Study.

*I am, Sir,* &c.

H. P.

# POST-SCRIPT.

ABout a Week after I had fent you the Letter, containing my Obfervations on *Polenus*'s Experiment, I had the good Fortune to hear an excellent and learned Friend of yours, to whom you had been pleafed to fhew my Letter, give a very curious and weighty Argument to confirm Sir *Ifaac Newton*'s Sentiment in relation to the Refiftance of Fluids, which I had deduced from the above mentioned Experiment; and as this very much pleafed me, I fhall here endeavour to fend you an Account of it in the following manner:

Suppofe pieces of fine Silk, or the like thin Subftance, extended in Parallel Planes, and fixt at fmall Diftances from each other. Suppofe then a Globe to ftrike perpendicularly againft the middle of the outermoft of the Silks, and by breaking through them to lofe part of its Motion. If the pieces of Silk be of equal Strength, the fame Degree of force will be required to break each of them; but the Time, in which each piece of Silk refifts, will be fo much fhorter as the Globe is fwifter; and the lofs of Motion in the Globe confequent upon its breaking through each Silk, and furmounting the Refiftance thereof, will be proportional to the Time, in which the Silk oppofes itfelf to the Globe's Motion; infomuch that the Globe by the Refiftance of any one piece of Silk will lofe fo much lefs of its Motion as it is fwifter. But on the other Hand, by how much fwifter the Globe moves,

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fo many more of the Silks it will break through in a given Space of Time; whence the number of the Silks, which oppofe themfelves to the Motion of the Globe in a given Time, being reciprocally proportional to the Effect of each Silk upon the Globe, the Refiftance made to the Globe by thefe Silks, or the lofs of Motion, the Globe undergoes by them in a given Time, will be always the fame.

Now if the Tenacity of the Parts of Fluids obferves the fame Rule as the Cohefion of the Parts of thefe Silks; namely, That a certain Degree of force is required to feparate and difunite the adhering Particles, the Refiftance arifing from the Tenacity of Fluids muft obferve the fame Rule as the Refiftance of the Silks, and therefore in a given Time the lofs of Motion, a Body undergoes in a Fluid by the Tenacity of its Parts, will in all Degrees of Velocity be the fame ; or in fewer Words, that part of the Refiftance of Fluids, which arifes from the Cohefion of their Parts, will be Uniform.

III. *An*